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QCD flux tubes for SU(3)

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We review the physical basis of dual QCD and solve the equations describing a QCD SU(3) flux tube. We use the solution to elucidate the physical connection between QCD and dual superconductivity.

I. INTRODUCTION

Understanding the physics of QCD at long range requires solving a strongly coupled gauge theory, since the usual gauge potential A_μ becomes singular at long range, and there is no known analytic approximation procedure available for calculating the correlation functions of A_μ in this regime. Nevertheless physical quantities exhibit a smooth and nonsingular behavior in a confining theory.

There is, in a gauge theory, however, the alternate possibility of using dual potentials to describe the long-distance regime. If we denote the Yang-Mills coupling constant by e so that $\alpha_s = e^2/4\pi$, then the coupling constant for the dual potential is $g = 2\pi/e$. The dual potentials are weakly coupled at long range and hence should be better variables in terms of which to describe long-distance physics.

The Yang-Mills Lagrangian, which in principle can be expressed in terms of dual potentials,[1] is invariant under dual gauge transformations of the dual potentials C_μ, given by

$$C_\mu \to \Omega^{-1} C_\mu \Omega + \frac{i}{g} \Omega^{-1} \partial_\mu \Omega \ , \tag{1.1}$$

where Ω, for N colors, is an SU(N) matrix. However, the transformation $A_\mu \to C_\mu$ is not explicitly known, and consequently one does not in practice know how to express the Yang-Mills Lagrangian in terms of dual potentials. In other words, "dual Yang-Mills" theory (i.e., the Yang-Mills Lagrangian as a function of the dual variables C_μ) cannot be explicitly written down.

We are interested, however, in solving Yang-Mills theory only in the long-distance regime. For this purpose it is only necessary to find the Lagrangian describing long-distance Yang-Mills dynamics in terms of the dual potentials. Let us denote this Lagrangian by $\mathcal{L}(C)$. Since we expect that the C_μ description of long-distance Yang-Mills dynamics is nonsingular, and that the C_μ fields interact weakly, $\mathcal{L}(C)$ should be a minimal dual gauge-invariant extension[2] of a quadratic Lagrangian $\mathcal{L}^{(0)}(C)$. Nonminimal additions to $\mathcal{L}(C)$ should not be relevant at long distances.

The first step,[2] then, is to construct the quadratic term $\mathcal{L}^{(0)}(C)$ of the long-distance Lagrangian $\mathcal{L}(C)$. The quadratic term describes a relativistic Abelian gauge theory characterized by a momentum-dependent magnetic permeability $\mu(q^2)$ which is the inverse of the dielectric constant $\epsilon(q^2)$ appropriate to the Abelian part of the conventional Yang-Mills gauge theory with which we began. In order to specify $\mu(q^2)$ we must have some information about long-distance ordinary Yang-Mills dynamics. During the past ten years a number of authors[3-10] have calculated $\epsilon(q^2)$ in the simplest self-consistent truncation of the Schwinger-Dyson equations of Yang-Mills theory compatible with the requirements of gauge invariance. These calculations have been carried out in different gauges and differ in technical details; nevertheless they all give a solution for $\epsilon(q^2)$ which as $q^2 \to 0$ has the behavior

$$\epsilon(q^2) \to -\frac{q^2}{M^2}, \quad q^2 \to 0 \ . \tag{1.2}$$

(Here M^2 is an undetermined parameter with the dimensions of a mass squared, and represents some of the unknown influence of short distances on long distances.)

The result (1.2) is certainly not an exact consequence of Yang-Mills theory. In fact, the corrections to this simplest self-consistent truncation of the Schwinger-Dyson equations are large at long range and are essentially uncalculable. This is a reflection of the unsuitability of the A_μ description of long-distance Yang-Mills theory.

Our basic assumption is that the long-distance expression (1.2) for $\epsilon(q^2)$ can be used to give the magnetic permeability $\mu(q^2)$, thus determining the quadratic part $\mathcal{L}^{(0)}(C)$ of the long-distance Lagrangian $\mathcal{L}(C)$ in the C_μ description. We therefore have

$$\mu(q^2) = -\frac{M^2}{q^2} + 1 \ , \tag{1.3}$$

where the choice of the constant 1 for the nonleading contribution to $\mu(q^2)$ represents a choice of normalization of the fields. Eq. (1.3) tells us that

$$\mathcal{L}^{(0)}(C)=-\tfrac{1}{4}(\partial_\sigma C_\nu-\partial_\nu C_\sigma)\mu(\partial_\sigma C_\nu-\partial_\nu C_\sigma)\ , \qquad (1.4)$$

where $\mu=1+M^2/\partial^2$. This Lagrangian produces a C_σ propagator Δ_C^0 of the form[11]

$$\Delta_C^0(q)\sim\frac{1}{q^2-M^2}\ . \qquad (1.5)$$

The next step is to construct the full long-distance Lagrangian $\mathcal{L}(C)$ as the minimal gauge-invariant extension of $\mathcal{L}^{(0)}(C)$ [we write the explicit form for $\mathcal{L}(C)$ below].

The self-consistency of the above procedure can be checked by using $\mathcal{L}(C)$ to calculate the exact long-distance C_μ propagator $\Delta_C(q)$. We find that aside from mass and wave-function-renormalization effects, $\Delta_C(q)$ has the same long-distance behavior as $\Delta_C^0(q)$. Furthermore, the effect of nonminimal additions to $\mathcal{L}(C)$ can be included at long distances by a redefinition of the coupling constants in $\mathcal{L}(C)$. This means that use of the expression (1.3) for $\mu(q^2)$ as the first step in obtaining the long-distance behavior of Yang-Mills theory in the C_μ description yields controllable results which are fixed by gauge invariance and which do not produce (other than through renormalization) changes in the starting point, Eq. (1.5).

Our view is that the impossibility of using (1.2) as a first approximation to long-distance Yang-Mills theory reflects the inadequacy of the A_μ description in that regime. Instead, by using its transcription (1.3) as the first approximation to long-distance Yang-Mills theory in the C_μ description, we have found a procedure for calculating physical processes in that domain. [We should emphasize that the relation $\epsilon\mu=1$, valid only in an Abelian theory, has been only used to find $\mathcal{L}^{(0)}(C)$. Hence we cannot use this relation to determine $\epsilon(q)$ from the full calculation of the long-distance behavior of $\mu(q)$ including the corrections imposed by gauge invariance in the dual description.]

Next let us write down the explicit form of $\mathcal{L}(C)$ obtained from the above procedure. It is the simplest possibility for $\mathcal{L}(C)$, incorporating the solution (1.2) of truncated Yang-Mills dynamics with the imposition of gauge invariance under the non-Abelian transformation (1.1), and it is the only known candidate for $\mathcal{L}(C)$. Further understanding requires detailed study of the quantum field theory based on $\mathcal{L}(C)$, which we have begun to do.

To write down $\mathcal{L}(C)$ we must first identify the dynamical variables. Because we start with a Lagrangian describing a medium with magnetic permeability $\mu(q)=1-M^2/q^2$, $\mathcal{L}$ will be a nonlocal function of the dual potentials C_μ^a, where a is the SU(N)-color index. It thus contains additional degrees of freedom which must be made explicit in order to write $\mathcal{L}$ in local form. These are represented by a set of tensor fields $\tilde{F}^a_{\mu\nu}=-\tilde{F}^a_{\nu\mu}$ and a set of ghost fields ψ_i^a and $\psi_i^{a\dagger}$, where the index i runs from one to three. We also introduce a gauge-fixing term $-(\lambda/2)(\partial_\mu C^{\mu a})^2$ appropriate to a generalized Landau gauge and the corresponding Faddeev-Popov ghosts χ^a.

The explicit expression for $\mathcal{L}(C)$ in terms of these variables is

$$\mathcal{L}(C)=\frac{M}{2}\tilde{F}_{\mu\nu}G^{\mu\nu}+\tfrac{1}{4}\tilde{F}_{\mu\nu}\mathcal{D}^2\tilde{F}^{\mu\nu}-\tfrac{1}{4}G_{\mu\nu}G^{\mu\nu}+\psi_i^\dagger(\mathcal{D}^2)\psi_i+\chi^\dagger\partial^\mu\mathcal{D}_\mu\chi-\frac{\lambda}{2}(\partial_\mu C_\mu)^2\ , \qquad (1.6)$$

where $\mathcal{D}_\mu$ is the total covariant derivative given by

$$(\mathcal{D}_\mu)_{ac}=\delta_{ac}\partial_\mu+gf_{abc}C_\mu^b\ , \qquad (1.7)$$

and $G^a_{\mu\nu}$ is the dual field tensor constructed from the potentials C_μ^a by the equation

$$G^a_{\mu\nu}=\partial_\mu C_\nu^a-\partial_\nu C_\mu^a+gf_{abd}C_\mu^bC_\nu^d\ . \qquad (1.8)$$

To see that (1.6) is the desired Lagrangian we form $\exp(i\int dx\,\mathcal{L})$ and carry out the quadratic integration over the variables $\tilde{F}^a_{\mu\nu}$, $\psi_i^{a\dagger}$, and ψ_i^a. As a result, the terms in $\mathcal{L}(C)$ depending upon these variables are replaced by

$$-\tfrac{1}{4}G^a_{\mu\nu}\left[\frac{M^2}{\mathcal{D}^2}\right]^{ab}G^{\mu\nu b}\ ,$$

which precisely corresponds to the non-Abelian generalization of a dielectric medium with permeability

$$\mu(q)=-\frac{M^2}{q^2}\ .$$

It is useful to introduce three-dimensional notation for the tensors $G_{\mu\nu}$ and $\tilde{F}_{\mu\nu}$. We define the color-electric displacement vector $\mathbf{D}^a$ and the color-magnetic $\mathbf{H}^a$ field by the dual equations

$$D_k^a\equiv\tfrac{1}{2}\epsilon_{klm}G^a_{lm},\quad H_k^a\equiv G^a_{0k}\ . \qquad (1.9)$$

Equations (1.8) and (1.9) then give $\mathbf{D}$ and $\mathbf{H}$ in terms of the potentials C_μ. Correspondingly, we define color-electric and -magnetic field vectors as $\mathbf{E}^a$ and $\mathbf{B}^a$, respectively, by the equations

$$E_k^a=-\tfrac{1}{2}\epsilon_{klm}\tilde{F}^a_{lm},\quad B_k^a=-\tilde{F}^a_{0k}\ . \qquad (1.10)$$

The variables $\mathbf{E}$ and $\mathbf{B}$ are independent of the dual potentials C_μ and serve as a convenient relabeling of the components of $\tilde{F}^a_{\mu\nu}$. The constitutive equations relating $\mathbf{D}$ and $\mathbf{H}$ to $\mathbf{E}$ and $\mathbf{B}$ follow as equations of motion obtained by varying $\tilde{F}_{\mu\nu}$ in the Lagrangian (1.6). It is these equations which justify the identification of $\mathbf{E}$ and $\mathbf{B}$ as color-electric and -magnetic fields.

Usual power-counting and gauge-invariance arguments show that the only divergences requiring additional counterterms in $\mathcal{L}$ arise from graphs containing either two or four external $\tilde{F}_{\mu\nu}$ or ψ_i^a lines. To satisfy the requirement of renormalizability, we therefore make the replacement

$$\mathcal{L}\to\mathcal{L}-W(\tilde{F},\psi,\psi^\dagger)\equiv\mathcal{L}\ , \qquad (1.11)$$

where $W(\tilde{F},\psi,\psi^\dagger)$ is a fourth-order polynomial in $\tilde{F}$, ψ, and $\psi^\dagger$. In this paper, where we discuss the SU(3) flux tube in the classical approximation, we will only need the terms in $W(\tilde{F},\psi,\psi^\dagger)$ which depend only upon $\tilde{F}$. This contribution to $W(\tilde{F},\psi,\psi^\dagger)$, which we denote as $W(\tilde{F})$, has the form

$$W(\tilde{F})=-\frac{\mu^2 N}{4}\tilde{F}^2+\frac{N\lambda}{4}W_4(\tilde{F})\ , \tag{1.12}$$

where

$$\tilde{F}^2=\tilde{F}^a_{\mu\nu}\tilde{F}^{\mu\nu a}=-2[(\mathbf{B}^a)^2-(\mathbf{E}^a)^2]\ , \tag{1.13}$$

and where $W_4(\tilde{F})$ is a quartic function of $\tilde{F}^a_{\mu\nu}$ whose explicit color and Lorentz structure is given in I (see Ref. 2). The parameter μ^2 has the dimension of a mass squared and λ is dimensionless. These new parameters represent the remaining effects of short-distance physics not already incorporated in the parameter M^2. From Eqs. (1.6), (1.11), and (1.12), we see that the fields $\tilde{F}^a_{\mu\nu}$ play the role of Higgs fields and $W(\tilde{F})$ that of the Higgs potential. We emphasize that these fields appear automatically once $\mathcal{L}(C)$ is written in local form and the "potential" W arises from renormalization effects. For stability we must have

$$\lambda>0\ . \tag{1.14}$$

Because of (1.12) and (1.14), the minimum of W always occurs at a nonzero value $\tilde{F}_{0\mu\nu}$ of $\tilde{F}_{\mu\nu}$:

$$\frac{\delta W}{\delta\tilde{F}_{\mu\nu}}=0 \quad \text{at } \tilde{F}_{\mu\nu}=\tilde{F}_{0\mu\nu}\ . \tag{1.15}$$

The value of W at this minimum is the vacuum energy density in the classical approximation. Using (1.12), (1.15), and the explicit tensor structure of $\tilde{F}_{0\mu\nu}$, we find

$$\epsilon_{\text{vac}}\equiv W(\tilde{F}_{0\mu\nu})=-\tfrac{1}{9}\lambda(\tilde{F}_0^2)^2\ . \tag{1.16}$$

Second, the trace anomaly for SU(N) Yang-Mills theory gives the relation[12]

$$\epsilon_{\text{vac}}=-\frac{11NG_2}{96}\ , \tag{1.17}$$

where G_2 is the gluon condensate. The value of G_2, determined by applying QCD sum rules in heavy-quark–antiquark systems, is approximately[12]

$$G_2\sim(330\ \text{MeV})^4\ . \tag{1.18}$$

Equations (1.16) and (1.17) give

$$\lambda=\frac{33}{32}\frac{NG_2}{(\tilde{F}_0^2)^2}\ . \tag{1.19}$$

The QCD vacuum is magnetic ($-\tilde{F}_0^2>0$), which, from Eq. (1.12), implies that μ^2 is negative. This spontaneous symmetry breaking, namely, $\tilde{F}_0^2\neq0$, present at zero temperature, disappears at a finite temperature T_c, and, therefore, so does confinement.[13]

Up to now we have discussed the origin and meaning of the quantities in the Lagrangian $\mathcal{L}(C)$ describing long-distance SU(N) pure Yang-Mills theory in terms of electric vector potentials C_μ. We must next add quarks. Quarks couple to the dual potentials C_μ in a manner analogous to the coupling of photons to magnetic monopoles. The interaction between the dual potentials and quarks therefore necessarily involves strings. This greatly complicates and severely limits the applications we can make to problems involving quarks. In fact Mandelstam's definition of the dual potential C_μ in terms of A_μ is no longer applicable in the presence of dynamical quarks. However, we can treat quarks approximately and show[14] that chiral symmetry is spontaneously broken provided we are in the confining vacuum $\tilde{F}_0^2\neq0$.

It is essential to understand the consistency of the quantum theory described by the Lagrangian $\mathcal{L}(C)$. Because of the Lorentz metric, the kinetic energy term in $\mathcal{L}(C)$ involving the fields $\tilde{F}^a_{\mu\nu}$ has the wrong sign for the fields $\mathbf{E}^a$. This might lead to a violation of unitarity if the fields $\mathbf{E}^a$ corresponded to physical degrees of freedom. We have not yet worked out the quantization procedure for $\mathcal{L}(C)$ in sufficient detail to determine whether this will in fact become a problem. However, when $M=0$, the theory possesses an additional symmetry, as a consequence of which the contribution arising from $\tilde{F}_{\mu\nu}$ internal lines to graphs containing only C_μ external lines cancels a corresponding contribution from internal ψ_i lines. There are then no physical degrees of freedom corresponding to any of the $\tilde{F}_{\mu\nu}$ fields. Amplitudes involving only C_μ quanta are determined by the pure Yang-Mills term $-\frac{1}{4}G_{\mu\nu}G^{\mu\nu}$ in Eq. (1.6) coming from the nonleading contribution to the permeability $\mu(q^2)$ in Eq. (1.3). One can, of course, arrive at this result directly by setting $M=0$ in the nonlocal term (1.9) of the dual Lagrangian.

However, the cancellation of the $\tilde{F}_{\mu\nu}$ and the ψ contributions to C_μ amplitudes when $M=0$ occurs only in the expansion about the perturbative vacuum, in which $\tilde{F}_{\mu\nu}=\psi=0$. It does not occur term by term in the expansion about the nonperturbative vacuum where $\tilde{F}_{\mu\nu}=\tilde{F}_{0\mu\nu}\neq0$. In fact when $\tilde{F}_0^2\neq0$, we will see that there exist classical flux-tube solutions when $M=0$ and that these solutions do not differ qualitatively from classical solutions previously obtained[2] for $M\neq0$. This indicates the nonperturbative $M=0$ solution is distinct from the perturbative $M=0$ result discussed in the previous paragraph. We are now studying the expansion about the nonperturbative vacuum. We are also studying the quantization of $\mathcal{L}(C)$ when $M\neq0$ in order to find the physical degrees of freedom. These are expected to be different from those of the perturbative $M=0$ theory, for which the $\tilde{F}_{\mu\nu}$ and ψ fields constitute unphysical degrees of freedom.

There is a second problem involving the fields $\mathbf{E}$. Because the classical nonperturbative vacuum is characterized by a nonvanishing expectation value $\tilde{F}_{0\mu\nu}$ of a tensor field $\tilde{F}_{\mu\nu}$ there is a potential violation of both rotational invariance and Lorentz-boost invariance. The spontaneous breakdown of rotational invariance is only apparent since the change in the vacuum under spatial rotations can be compensated for by a gauge transformation. The spontaneous breakdown of Lorentz-boost invariance in the classical approximation leads to a set of negative metric Goldstone particles associated with certain components of the fields $\mathbf{E}$. Hence, understanding potential violations of Lorentz invariance is directly connected to understanding the degrees of freedom corresponding to the fields $\mathbf{E}$. The consistent quantization of $\mathcal{L}(C)$, Eq. (1.11), is the central problem remaining to be solved in order to put dual QCD on a firm basis, and it is this problem on which we are now working.

In the present paper we study the classical SU(3) field equations generated by $\mathcal{L}(C)$ and find a flux-tube solution to SU(3) dual Yang-Mills theory. We then find the string tension σ by calculating the energy per unit length of this SU(3) flux tube.

II. THE FLUX-TUBE EQUATIONS FOR SU(3)

We begin our discussion by determining the behavior of the fields at large distances ρ from the center of an SU(N) flux tube. (We choose the z axis to be the axis of the flux tube and use cylindrical coordinates ρ, ϕ, and z.) At large ρ the dual potential is a pure gauge,

$$C_\mu \rightarrow \frac{i}{g}\Omega^{-1}\partial_\mu \Omega \ , \tag{2.1}$$

and the corresponding $\tilde{F}_{\mu\nu}$ field is a gauge transformation of the (constant) vacuum field $\tilde{F}_{0\mu\nu}$. Thus

$$\tilde{F}_{\mu\nu} \rightarrow \Omega^{-1}\tilde{F}_{0\mu\nu}\Omega \ . \tag{2.2}$$

We know that the nonperturbative vacuum is magnetic, so we can choose the electric fields in $\tilde{F}_{0\mu\nu}$ to vanish, and only the magnetic fields $\mathbf{B}_0$ exist. Our ansatz for these will be

$$\mathbf{B}_0 = b(\hat{\mathbf{e}}_x J_x + \hat{\mathbf{e}}_y J_y + \hat{\mathbf{e}}_z J_z), \quad \mathbf{E}_0 = 0 \ , \tag{2.3}$$

where b is a constant determined by $\tilde{F}_0^2$, and the three J's are color matrices in an N-dimensional irreducible representation of the generators of SU(2). They satisfy the relation

$$J_x^2 + J_y^2 + J_z^2 = \frac{N^2-1}{4} \ . \tag{2.4}$$

The form (2.3) automatically ensures rotational invariance of the asymptotic solution, in that a spatial rotation can always be compensated for by a gauge transformation. Using Eqs. (2.3) and (2.4) we obtain

$$b^2 = -\tilde{F}_0^2/N(N^2-1) \ . \tag{2.5}$$

For the gauge transformation Ω, our ansatz will be

$$\Omega = e^{-inY\varphi}, \quad n = 0, 1, \ldots, N-1 \ , \tag{2.6}$$

where we choose the traceless $N\times N$ SU(N) matrix Y so that

$$e^{-2\pi i n Y} = e^{-2\pi i n/N} \ , \tag{2.7}$$

in order to guarantee single valuedness.[2] The corresponding solution is a Z_N vortex carrying n units of quantized Z_N flux.

We have seen in our numerical study[2] of the SU(2) flux-tube equations that the fields C_0 and $\mathbf{E}$ which are zero asymptotically remain small at all distances and do not contribute substantially to the string tension. The coupling of C_0 and $\mathbf{E}$ to $\mathbf{B}$ and $\mathbf{C}$ arises from the $(M/2)\tilde{F}_{\mu\nu}G^{\mu\nu}$ term in the Lagrangian (1.6). The relative insensitivity of the string tension σ to C_0 and $\mathbf{E}$ reflects the fact that the relevant scale determining σ is $-\tilde{F}_0^2$ derived from W by Eq. (1.15) rather than M. In our study of the SU(3) flux-tube equations we will set $M=0$. This greatly simplifies the equations which now involve only $\mathbf{B}$ and $\mathbf{C}$. Based on our study of the SU(2) case, this simplification will not change our results qualitatively.

We make the following ansatz for the color structure of the components of $\mathbf{B}$ and $\mathbf{C}$ in cylindrical coordinates:

$$B_\rho = B_1(\rho) b_\rho, \quad B_\phi = B_2(\rho) b_\phi, \quad B_3 = B_3(\rho) b_z \ , \tag{2.8}$$

$$C_\phi = C(\rho) Y, \quad C_\rho = C_z = 0 \ , \tag{2.9}$$

where

$$b_\rho = \Omega^{-1}(J_x \cos\phi + J_y \sin\phi)\Omega \ , \tag{2.10a}$$

$$b_\phi = \Omega^{-1}(-J_x \sin\phi + J_y \cos\phi)\Omega \ , \tag{2.10b}$$

$$b_z = \Omega^{-1} J_z \Omega \ , \tag{2.10c}$$

and where, for SU(3),

$$J_x = \lambda_7, \quad J_y = -\lambda_5, \quad J_z = \lambda_2, \quad Y = \frac{1}{\sqrt{3}}\lambda_8 \ . \tag{2.11}$$

Then from Eqs. (2.1)–(2.3), (2.6), (2.8), and (2.9) we obtain

$$\lim_{\rho\to\infty} B_i(\rho) = b, \quad i = 1, 2, 3 \ , \tag{2.12a}$$

$$\lim_{\rho\to\infty} C(\rho) = -\frac{n}{\rho g} \ . \tag{2.12b}$$

For the case of SU(3) the ansatz (2.8) and (2.9) with $B_1(\rho) = B_2(\rho) = B(\rho)$ closes. We then set

$$B_1(\rho) = B_2(\rho) = B(\rho) \ , \tag{2.13a}$$

and use Eqs. (2.8) and (2.10) to obtain

$$\mathbf{B}(\mathbf{x}) = \Omega^{-1}[B(\rho)(J_x\hat{\mathbf{e}}_x + J_y\hat{\mathbf{e}}_y) + B_3(\rho)J_z]\Omega \ . \tag{2.13b}$$

If $B_1(\rho) \neq B_2(\rho)$ one must add additional terms to (2.8) and (2.9) to close the equations. For SU(N) even with $B_1 = B_2$ the ansatz (2.8) and (2.9) does not close. In the following we will for the most part restrict ourselves to SU(3) for which Eqs. (2.9) and (2.13) allow us to obtain an exact solution. However, we will carry out our treatment so that it is easily generalizable to SU(N).

Elementary dimensional arguments show that there are two natural scales in $\mathcal{L}$: an energy scale $(-\tilde{F}_0^2)^{1/2}$ to which the square root of the string tension is proportional, and a length scale $1/(-\lambda\tilde{F}_0^2)^{1/2}$, which characterizes the radius of the electric-color flux tubes. This fact is made manifest by introducing dimensional variables based on these scales as follows:

$$\rho' = (-\lambda\tilde{F}_0^2)^{1/2}\rho \ , \tag{2.14a}$$

$$\mathbf{C}' = \frac{1}{(-\tilde{F}_0^2)^{1/2}}\mathbf{C} \ , \tag{2.14b}$$

$$\mathbf{B}' = \frac{1}{(-\tilde{F}_0^2)^{1/2}}\mathbf{B}\left[\frac{N(N^2-1)}{6}\right]^{1/2} \ . \tag{2.14c}$$

The factor $[N(N^2-1)]/6$ is introduced into Eq. (2.14c) so that

$$\lim_{\rho'\to\infty} B_i'(\rho')=\sqrt{1/6}\ , \tag{2.15}$$

independent of N. [See Eqs. (2.5) and (2.12a).]

Using Eqs. (2.8), (2.10), (2.14c), (1.12), and the explicit form of $W_4(\tilde{\rho})$ given in (I), we obtain the following expression for $W(\tilde{F})$:

$$W(\tilde{F})=-\lambda(\tilde{F}_0^2)^2 W'\ , \tag{2.16a}$$

where

$$W'=-\frac{1}{9}\left[4(B_1'^2+B_2'^2+B_3'^2)-\frac{3}{5}\left[\frac{(4N^2-1)(B_1'^2+B_2'^2+B_3'^2)^2+(8N^2-17)(B_1'^4+B_2'^4+B_3'^4)}{N^2-1}\right]\right]\ . \tag{2.16b}$$

Using the particular ansatz (2.13a) and setting $N=3$, we can write W' as

$$W'=-\tfrac{1}{9}\{4(2B'^2+B_3'^2)-\tfrac{1}{8}[21(2B'^2+B_3'^2)^2+33(2B'^4+B_3'^4)]\}\ . \tag{2.17}$$

Inserting the SU(3) ansatz (2.9), and (2.13) into the Lagrangian $\mathcal{L}$ and defining

$$g'=\frac{g}{\sqrt{\lambda}}\ , \tag{2.18}$$

we obtain

$$\mathcal{L}=-\lambda(\tilde{F}_0^2)^2\left[\tfrac{2}{3}C'\tilde{\nabla}'^2C'+B'\nabla'^2B'-g'^2B'^2\left[C'+\frac{n}{\rho'g'}\right]^2+\tfrac{1}{2}B_3'\nabla'^2B_3'-W'\right]\ , \tag{2.19}$$

where

$$\nabla^2=\frac{1}{\rho}\frac{d}{d\rho}\rho\frac{d}{d\rho}\quad\text{and}\quad\tilde{\nabla}^2=\nabla^2-\frac{1}{\rho^2}\ .$$

The equations of motion generated by (2.19) are

$$\tfrac{4}{3}\tilde{\nabla}'^2C'-2g'^2B'^2\left[C'+\frac{n}{\rho'g'}\right]=0\ , \tag{2.20a}$$

$$\nabla'^2B'-g'^2\left[C'+\frac{n}{\rho'g'}\right]^2=\frac{1}{2}\frac{\delta W'}{\delta B'}\ , \tag{2.20b}$$

$$\nabla'^2B_3'=\frac{\delta W'}{\delta B_3'}\ . \tag{2.20c}$$

Finally the string tension σ (the energy per unit length) is given in terms of the Hamiltonian density $\mathcal{H}=-\mathcal{L}$ as

$$\sigma=2\pi\int_0^\infty \rho\, d\rho(-\mathcal{L})\ . \tag{2.21}$$

Using Eqs. (2.19), (2.20), and (2.21) we obtain

$$\sigma=(-\tilde{F}_0^2)\sigma_d\ , \tag{2.22a}$$

where

$$\sigma_d\equiv\sigma_d(g'^2)=2\pi\int_0^\infty\rho' d\rho'\left[W'-\frac{1}{2}B'\frac{\delta W'}{\delta B'}-\frac{1}{2}B_3'\frac{\delta W'}{\delta B_3'}-g'^2B'^2C'\left[C'+\frac{n}{\rho'g'}\right]+\frac{1}{9}\right]\ . \tag{2.22b}$$

The term $(+\frac{1}{9})$ in Eq. (2.22b) comes from subtracting the vacuum energy density Eq. (1.16) from $\mathcal{H}$.

III. SOLUTIONS OF THE SU(3) FLUX-TUBE EQUATIONS AND COMPARISON WITH THE LANDAU-GINZBURG EQUATIONS

The dimensionless string tension σ_d depends upon the single parameter g'. From Eqs. (1.19) and (2.18) we obtain

$$g'^2=\frac{32g^2(\tilde{F}_0^2)^2}{33NG_2}\ . \tag{3.1}$$

The coupling constant g^2 can be estimated from the $1/R$ contribution to the phenomenologically determined potential between heavy quarks.[15] This Coulomb-like contribution is

$$V_c(R)=-\frac{4}{3}\frac{\alpha_s}{R}=-\frac{4}{3}\frac{\pi}{g^2R}\ . \tag{3.2}$$

The fit of Ref. 15 then gives

$$g^2\sim 6.3\ ; \tag{3.3}$$

the estimate (3.3) corresponds to the effective coupling at distance scales $R\sim(1\ \text{GeV})^{-1}$. Combining Eqs. (2.22a) and (3.1) yields

$$\frac{1}{g'}=\left[\frac{33NG_2}{32g^2\sigma^2}\right]^{1/2}\sigma_d(g'^2)\ . \tag{3.4}$$

Now in order to compare our results with those of superconductivity it is convenient to define a parameter κ given by

$$\kappa=\frac{5}{3g'}=\frac{5\sqrt{\lambda}}{3g}\ . \tag{3.5}$$

The connection with superconductivity is made by setting B_3' equal to its asymptotic value $\sqrt{1/6}$ in Eq. (2.20b). Equations (2.20a) and (2.20b) then become the Landau-Ginzburg equations of the Abelian Higgs[16] model with

Landau-Ginzburg parameter κ given by Eq. (3.5). Since we will see that the exact solution of Eqs. (2.20) yields a $B_3'(\rho')$ which does not change substantially from its asymptotic value, the value of σ_d obtained for our SU(3) flux tubes is close to the Landau-Ginzburg value. Thus qualitatively Eqs. (2.20) describe a dual superconductor with Landau-Ginzburg parameter κ. For this reason we will use this parameter to describe our results.

When expressed in terms of κ, Eq. (3.4) becomes [for SU(3)]

$$\kappa=\frac{5}{4}\left[\frac{11}{2g^2}\right]^{1/2}\frac{\sqrt{G_2}}{\sigma}\sigma_d(\kappa)\ . \tag{3.6}$$

With the estimates, (3.3) and (1.18) for g^2 and G_2 and the value

$$\sigma=(420\ \text{MeV})^2 \tag{3.7}$$

for the string tension, Eq. (3.6) reduces to

$$\sigma_d(\kappa)=1.4\kappa\ . \tag{3.8}$$

We have solved the SU(3) flux-tube equations (2.20) for $n=1$, for a range of values of g'^2, i.e., κ, and calculated the dimensionless string tension $\sigma_d(\kappa)$ from Eq. (2.22b). (This corresponds to an $n=1$ Z_3 vortex.) The resulting function $\sigma_d(\kappa)$ as well as the straight line Eq. (3.8) are plotted in Fig. 1. They intersect at the point

$$\kappa\approx\sqrt{5/9}=0.745\ , \tag{3.9a}$$

$$\sigma_d\approx 1.033\ . \tag{3.9b}$$

Equations (3.9a), (3.3), and (3.5) then give

$$\lambda=1.26\ , \tag{3.10}$$

while Eqs. (3.9b) and (2.22a) give

$$(-\tilde{F}_0^2)^{1/2}=413\ \text{MeV}\ . \tag{3.11}$$

Equations (3.10) and (3.11) are then the values of the fundamental parameters of dual QCD which reflect the effect of short-distance physics on long distances. Of course,

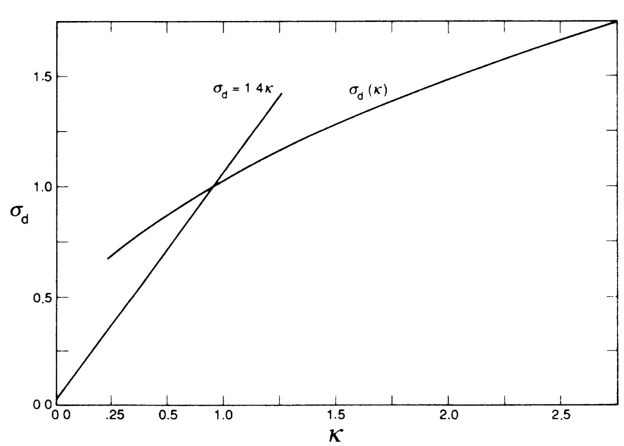

FIG. 1. The function $\sigma_d(\kappa)$ as a function of κ, compared to the straight line $\sigma_d(\kappa)=1.4\kappa$.

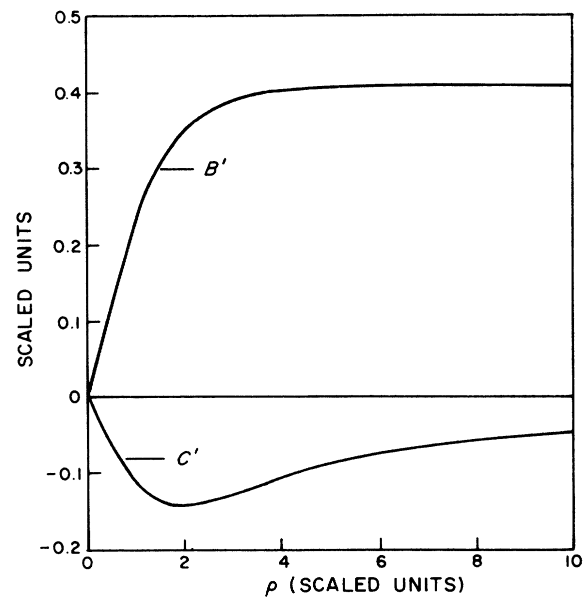

FIG. 2. The scaled vector potential C' and magnetic field B' plotted as functions of the scaled distance ρ'.

the uncertainties of a factor of 2 in G_2 as well as uncertainties in other quantities entering into Eq. (3.6) mean that Eqs. (3.10) and (3.11) are only semiquantitative.

We can next calculate the radius R_{FT} of the flux tube:

$$R_{\text{FT}}^2=\left[\frac{1}{-\lambda\tilde{F}_0^2}\right]\frac{\int\rho' d\rho'\rho'^2\mathcal{H}(\rho')}{\int\rho' d\rho'\mathcal{H}(\rho')}\ . \tag{3.12}$$

Using our solution to Eqs. (2.20) for $\kappa=0.745$, we find

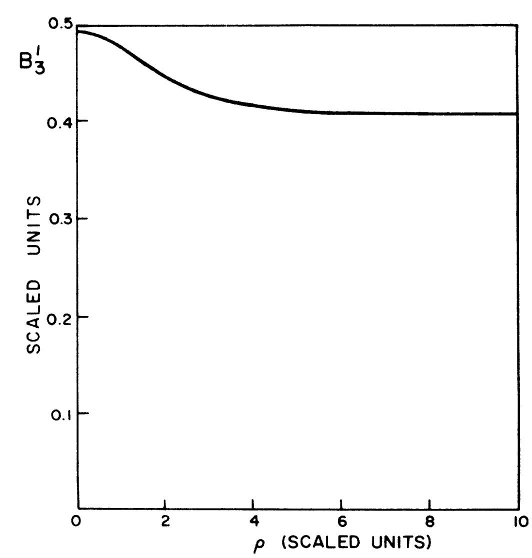

FIG. 3. The scaled magnetic field B_3' plotted as a function of ρ'.

$$R_{\mathrm{FT}}=0.95 \text{ fm} . \tag{3.13}$$

Thus the QCD flux tube has a radius of about 1 fm. This is reflected in the radial dependences of the explicit solutions of Eqs. (2.20). Next we note that the self-consistent value $\kappa=0.745$ of our solution is very close to $1/\sqrt{2}$, which in the Landau-Ginzburg equations is the critical value separating type-II superconductors, $\kappa>1/\sqrt{2}$, from type-I superconductors, $\kappa<1/\sqrt{2}$. We thus expect that C' will begin to decrease at values of ρ' for which B' has reached about one-half of its asymptotic value. We see this feature explicitly in Fig. 2 in which C' and B' are plotted as functions of ρ'. In Fig. 3 we see that B_3' is approximately constant taking on values between 0.4 and 0.5 which indicates that the color-electric vortex of QCD qualitatively has the structure of a magnetic vortex of superconductivity.

The fact that our solution for an infinitely long QCD flux tube is similar to the Nielsen-Olesen magnetic vortex of the Abelian Higgs model[17] suggests that the static potential $V^{q\bar{q}}(R)$ between a heavy-quark–antiquark pair separated by a finite distance R may be correspondingly similar to the potential $V^{m\bar{m}}(R)$ between a monopole-antimonopole pair separated by a finite distance in a superconductor having $\kappa=0.745$. Since the latter situation is described by an Abelian field theory, the classical calculation of $V^{m\bar{m}}(R)$ is unambiguous and has been carried out by Ball and Caticha.[18] At large R the potential is, of course, linear and is determined by the string tension. At short distances it has the Coulomb behavior, Eq. (3.2), with a strength determined by the monopole charge. (There is, of course, no color factor $\frac{4}{3}$ since the monopole charge is given exactly by the flux emanating from the monopole.) Furthermore, the exact $V^{m\bar{m}}(R)$ as calculated by Ball and Caticha is very well approximated at all distances by a sum of a Coulomb potential and a linear potential. With the correspondence

$$V^{q\bar{q}}(R) \Longrightarrow V^{m\bar{m}}(R)\big|_{\kappa=0.745} ,$$

we would then expect that the heavy-quark–antiquark potential could also be approximated by the sum of Coloumb and linear terms. This then provides a theoretical basis for the phenomenological $q\bar{q}$ potential of Ref. 15.

IV. SUMMARY AND CONCLUSIONS

We have solved a simplified version of the equations for the $n=1$ Z_3 flux tube in SU(3) dual QCD. A comparison of the string tension obtained from this solution with the experimentally determined string tension fixes the parameters of dual QCD. All remaining features of this SU(3) flux tube are then uniquely determined. In particular, the mean-squared radius is about 1 fm and the effective "Landau-Ginzburg parameter" characterizing the flux tube is close to the critical value separating type-I and type-II superconductors.

It remains to understand the physics of the simplification $M=0$ used to obtain this solution. This simplification decoupled the color-electric fields **E** from the flux-tube equations. Thus this question, like those discussed in Sec. II, is related to the degrees of freedom corresponding to the fields **E**. An ultimate understanding of these issues is thus essential to all aspects of dual QCD.

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